Sample Question Paper - 37
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours | ***Maximum Marks : 40***

General Instructions :

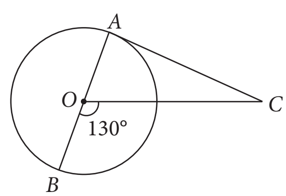
1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Solve the quadratic equation $9x^2 - 6b^2x - (a^4 - b^4) = 0$ for x.

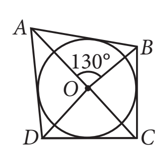
2. What will be the 21st term of the A.P. whose first two terms are –3 and 4?

3. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 130°$, then find $\angle ACO$.



OR

In the given figure, if $\angle AOB = 130°$, then find $\angle COD$.



4. If $a = -7$, $b = 12$ in $x^2 + ax + b = 0$, then find the smaller root of the given equation

5. If the mean of a, b, c is M and $ab + bc + ca = 0$, then the mean of a^2, b^2, c^2 is kM^2. Find the value of k.

6. The dimensions of a metallic cuboid are 100 cm × 80 cm × 64 cm. It is melted and recast into a cube. Find the edge of the cube.

OR

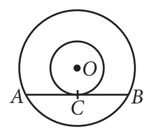
A cube of side 6 cm is cut into a number of cubes, each of side 2 cm. Find the number of cubes formed.

SECTION - B

7. Find the mode of the following data :

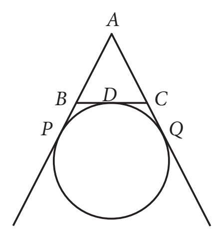
Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

8. In the given figure, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that $AC = CB$.



OR

In figure, find the perimeter of ΔABC, if $AP = 12$ cm.



9. Find the mean of the given distribution by direct method.

Class-interval	0-10	11-20	21-30	31-40	41-50
Frequency	3	4	2	5	6

10. A ladder of length 6 m makes an angle of 45° with the floor while leaning against one wall of a room. If the foot of the ladder is kept fixed on the floor and it is made to lean against the opposite wall of the room, it makes an angle of 60° with the floor. Find the distance between these two walls of the room.

SECTION - C

11. Ramkali required ₹2500 after 12 weeks to send her daughter to school. She saved ₹100 in the first week and increased her weekly saving by ₹20 every week. Find whether she will be able to send her daughter to school after 12 weeks.

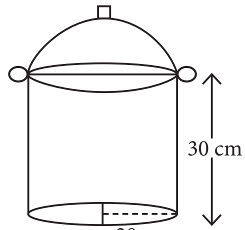
OR

The 16^{th} term of an A.P. is 1 more than twice its 8^{th} term. If the 12^{th} term of the A.P. is 47, then find its n^{th} term.

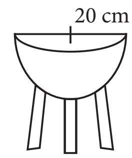
12. Draw a line segment of length 7 cm and divide it internally in the ratio 2 : 3.

Case Study - 1

13. In the evening, Mona and her mother went to ice-cream parlour to eat ice cream. Mona observe that ice cream seller has two different kinds of container as given below.



(Cylindrical container with hemispherical end)
(I)



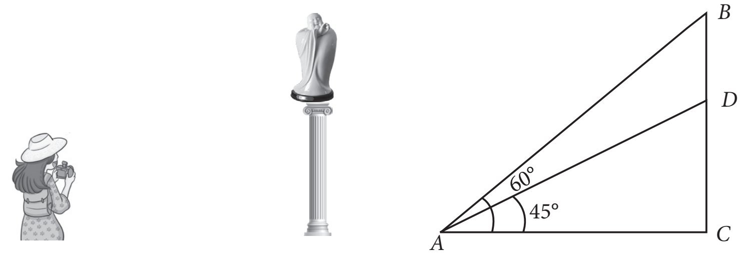
(Hemispherical container having three pillars at bottom)
(II)

(i) Find the total surface area of the type (I) container.

(ii) Find the volume of type-II container.

Case Study - 2

14. In an exhibition, a statue stands on the top of a pedestal. From the point on ground where a girl is clicking the photograph of the statue the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of pedestal is 45°.



Based on the above information, answer the following questions.

(i) If the height of the pedestal is 20 m, then find the height of the statue.

(ii) If the height of the statue is 1.6 m, then find the height of the pedestal.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. We have, $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$\Rightarrow 9x^2 - 6b^2x - a^4 + b^4 = 0$

$\Rightarrow \{(3x)^2 - 2(3x)b^2 + (b^2)^2\} - (a^2)^2 = 0$

$\Rightarrow (3x - b^2)^2 - (a^2)^2 = 0$

$\Rightarrow (3x - b^2 + a^2)(3x - b^2 - a^2) = 0$

$\Rightarrow 3x - b^2 + a^2 = 0$ or $3x - b^2 - a^2 = 0$

$\Rightarrow 3x = b^2 - a^2$ or $3x = b^2 + a^2$

$\Rightarrow x = \dfrac{b^2 - a^2}{3}$ or $x = \dfrac{a^2 + b^2}{3}$

2. Given, $a = -3$ and $a + d = 4$

$\Rightarrow -3 + d = 4 \Rightarrow d = 7$

$\therefore a_{21} = a + (21 - 1)d$ $[\because a_n = a + (n - 1)d]$

$= -3 + (20)7 = -3 + 140 = 137$

3. Given, $\angle BOC = 130°$

Since, AC is a tangent to the circle at A.

$\therefore \angle OAC = 90°$ [$\because$ Radius is perpendicular to the tangent at point of contact]

Now, $\angle AOC + \angle BOC = 180°$ [Linear pair]

$\Rightarrow \angle AOC = 180° - 130° = 50°$

In ΔAOC, $\angle AOC + \angle ACO + \angle OAC = 180°$

[Angle sum property]

$\Rightarrow \angle ACO = 180° - 50° - 90° = 40°$

OR

Since opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

$\therefore \angle AOB + \angle COD = 180°$

$\Rightarrow 130° + \angle COD = 180°$

$\Rightarrow \angle COD = 180° - 130° = 50°$

4. We have, $x^2 + ax + b = 0$, where $a = -7$, $b = 12$

$\therefore x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 4x - 3x + 12 = 0$

$\Rightarrow x(x - 4) - 3(x - 4) = 0$

$\Rightarrow (x - 4)(x - 3) = 0$

$\Rightarrow x = 4$ or $x = 3$

Thus, smaller root of the given equation is 3.

5. Given, mean of a, b and c is M.

$\therefore a + b + c = 3M$ ...(i)

Also, $ab + bc + ca = 0$ [Given] ...(ii)

Now, $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

$\Rightarrow a^2 + b^2 + c^2 = (3M)^2 - 2(0)$ [From (i) and (ii]

$\Rightarrow a^2 + b^2 + c^2 = 9M^2 - 0 = 9M^2$

$\therefore$ Mean of a^2, b^2 and $c^2 = \dfrac{a^2 + b^2 + c^2}{3} = \dfrac{9M^2}{3} = 3M^2$

6. Volume of given cuboid $= 100 \times 80 \times 64$

$= 512000 \text{ cm}^3$

Now, cuboid is melted and recast into a cube.

Let side of the cube = a cm

Also, volume of the cube = volume of the cuboid

$\Rightarrow a^3 = 512000 \Rightarrow a = 80$

Hence, edge of the cube is 80 cm.

OR

Number of cubes formed

$$= \frac{\text{Volume of given cube}}{\text{Volume of each small cube}} = \frac{6 \times 6 \times 6}{2 \times 2 \times 2} = 27.$$

7. From the given data, we observe that, highest frequency is 20, which lies in the class-interval 40-50.

$\therefore l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{20 - 12}{40 - 12 - 11}\right) \times 10$$

$$= 40 + \frac{80}{17} = 40 + 4.7 = 44.7$$

8. Given : Two concentric circles C_1 and C_2 with centre O and AB is the chord of C_1 touching C_2 at C.

To Prove : $AC = CB$

Construction : Join OC.

Proof : We know that, tangent at any point to the circle is perpendicular to the radius at the point of contact.

$\therefore OC \perp AB$ ($\because$ AB is tangent for C_2)

Since, perpendicular drawn from the centre to the chord bisects the chord.

$\therefore AC = CB$ ($\because$ AB is a chord for C_1)

OR

As we know that, tangents drawn from an external point are equal in length.

$\therefore$ $BP = BD$ and $CD = CQ$ ...(i)

Also, $AP = AQ = 12$ cm

$\Rightarrow$ $AB + BP = 12$ cm and $AC + CQ = 12$ cm

$\Rightarrow$ $AB + BD = 12$ cm and $AC + CD = 12$ cm ...(ii) [Using (i)]

Now, perimeter of $\Delta ABC = AB + BC + CA$

$= AB + BD + DC + AC$

$= 12 + 12$ [Using (ii)]

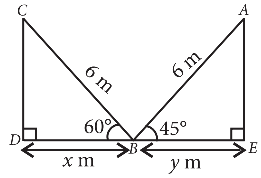
$= 24$ cm

9. Let us construct the following table for the given data.

Class-interval	Frequency (f_i)	Class mark (x_i)	$f_i x_i$
0 - 10	3	5.0	15.0
11 - 20	4	15.5	62.0
21 - 30	2	25.5	51.0
31 - 40	5	35.5	177.5
41 - 50	6	45.5	273.0
Total	$\Sigma f_i = 20$		$\Sigma f_i x_i = 578.5$

$$\therefore \text{ Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{578.5}{20} = 28.925$$

10. Let AB, CB be the ladder and AE, CD are the walls of the room.



Also let $BD = x$ m and $BE = y$ m

In ΔABE,

$$\cos 45° = \frac{BE}{AB} = \frac{y}{6} \Rightarrow y = 6 \times \frac{1}{\sqrt{2}} = 3\sqrt{2}$$

In ΔDBC, $\cos 60° = \frac{BD}{BC} = \frac{x}{6} \Rightarrow x = 6 \times \frac{1}{2} = 3$

Now, distance between the walls $= DE$

$$= x + y = 3 + 3\sqrt{2} = 3(1+\sqrt{2}) \text{ m}$$

11. Saving of first week = ₹100

Saving of second week = ₹100 + ₹20 = ₹120

Saving of third week = ₹120 + ₹20 = ₹140

So, 100, 120, 140, ... [forms an A.P.]

Here, $a = 100$ and $d = 120 - 100 = 20$, $n = 12$

$$\therefore S_{12} = \frac{12}{2}\{2 \times 100 + (12-1)20\}$$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$= 6\{200 + 220\} = 2520$

Since ₹2520 > ₹2500

So, she would be able to send her daughter to school after 12 weeks.

OR

Let a be the first term and d be the common difference of the A.P.

According to question, $a_{16} = 2a_8 + 1$

$\Rightarrow$ $a + 15d = 2[a + 7d] + 1 \Rightarrow a + 15d = 2a + 14d + 1$

$\Rightarrow$ $d = a + 1$ ...(i)

Also, $a_{12} = 47$ $\Rightarrow$ $a + 11d = 47$

$\Rightarrow$ $a + 11(a + 1) = 47$ [Using (i)]

$\Rightarrow$ $a + 11a + 11 = 47 \Rightarrow 12a = 36$ or $a = 3$

$\therefore$ $d = 3 + 1 = 4$

Now, n^{th} term of the A.P., $a_n = a + (n - 1)d$

$\therefore$ $a_n = 3 + (n - 1)\,4 = 3 + 4n - 4 = 4n - 1$

12. Steps of construction:

Step-I : Draw a line segment $AB = 7$ cm.

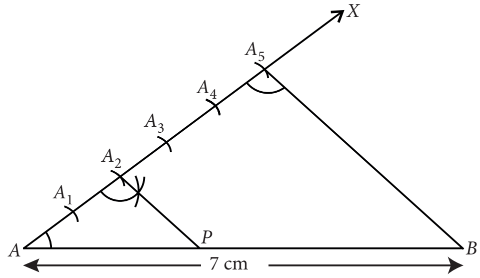
Step-II : Draw any ray AX making an acute angle with AB.

Step-III : On ray AX, mark $2 + 3 = 5$ points A_1, A_2, A_3, A_4, A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.

Step-IV : Join A_5B.

Step-V : From A_2, draw $A_2P \| A_5B$, meeting AB at P.

Thus P divides AB in the ratio 2 : 3.



13. (i) We have, $r = 20$ cm, $h = 30$ cm

Total surface area of type (I) container

$= 2\pi rh + \pi r^2 + 2\pi r^2$

$= 2\pi rh + 3\pi r^2 = 2\times\frac{22}{7}\times 20\times 30 + 3\times\frac{22}{7}\times 20\times 20$

$= \frac{22}{7}\times 20[60+60] = \frac{22\times 20\times 120}{7} = \frac{52800}{7}$

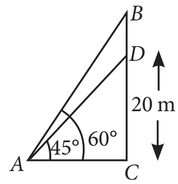
$= 7542.86$ cm^2

(ii) We have, $r = 20$ cm

Volume of type (II) container $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\frac{22}{7}\times 20\times 20\times 20 = \frac{352000}{21} = 16761.90$ cm^3

14. (i) In ΔACD,



$\tan 45° = \frac{CD}{AC} = 1$

$\therefore \quad AC = CD = 20$ m ...(i)

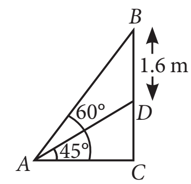
Let, $BD = h$ m be the height of the statue.

In ΔABC, $\tan 60° = \frac{BC}{AC} \Rightarrow \frac{BD+CD}{AC} = \sqrt{3}$

$\Rightarrow \frac{20+h}{20} = \sqrt{3}$ [using (i)] $\Rightarrow h = 20(\sqrt{3}-1)$ m.

(ii) Since, in ΔACD, $\angle DAC = 45°$

$\therefore \quad AC = CD$ (say x)



In ΔBAC, $\tan 60° = \frac{BC}{AC}$

$\Rightarrow \frac{1.6+x}{x} = \sqrt{3} \Rightarrow 1.6 = x(\sqrt{3}-1)$

$\Rightarrow \quad x = \frac{1.6}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1} = 0.8(\sqrt{3}+1)$ m